

Noise-robust Compressed Sensing Method for Super-resolution TOA Estimation

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Abstract: Super-resolution time of arrival estimation methods have attracted much attention in radar signal processing. Many studies have used compressed sensing (CS)-based approaches to attain the super-resolution property because they assume sparseness of the object temporal distribution. However, this approach still suffer from accuracy degradation when decomposing highly correlated signals in heavily noise-contaminated situations. To resolve this problem, this study introduces an enhanced CS method by exploiting the sparseness of both the time and frequency domains of the target signals. Numerical simulation and comparison with results obtained by conventional methods demonstrate that the proposed method considerably enhances the reconstruction accuracy for multiple highly correlated signals in lower signal-to-noise ratio situations.

Keywords: Super-resolution TOA estimation, Compressed sensing (CS), Radar signal processing, l_1 -norm minimization

Classification: Signal Processing

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1 Introduction

Microwave radar systems are used for detection or ranging tools in a wide variety of applications such as all-weather type terrain surface measurement and automobile or indoor sensing radar in optically harsh environments. However, owing to regulatory restrictions and hardware limitations, the frequency bands available for radar are severely limited. Therefore, super-resolution time of arrival (TOA) estimation approaches such as multiple signal classification (MUSIC) methods [1] have been intensively investigated in many previous studies. The MUSIC method exploits the orthogonality between the signal and noise eigenvector correlation matrix and achieves a higher TOA resolution than the Capon method or other TOA methods. However, the MUSIC method requires a priori knowledge of the number of targets and involves loss of scattering coefficient information[1]. Furthermore, these methods suffer from low resolution and accuracy when separating highly correlated signals that are typically found in multiple reflection echoes in radar applications. As another solution for this problem, this study focuses on the compressed sensing (CS) approach[2]. CS is widely recognized as a useful solution for underdetermined and ill-posed inverse problems with constrained l_1 norm minimization. It requires the simple assumption that the spatial or temporal distribution of targets should be sparse compared with the total sampled area [3]. There are intensive researches for CS-based signal processing for radar applications, which achieved both a relatively lower sampling rate and high-resolution property the TDOA (Time Difference of Arrival) discrimination issues [4]. However, it has been reported that the original CS algorithm suffers from inaccuracy in the case of strongly contaminated by noise, especially when highly correlated target signals are also closely located within the theoretical range resolution.

To retain the TOA accuracy and resolution under the situation that highly correlated signals are mixed together in lower SNR levels, this study introduces sparse regularization for the frequency domain in the original CS cost function. This regularization term prevents an over-fitting to noise component in the frequency domain, acting as a kind of a bandpass filter. Numerical simulation demonstrates that the proposed method retains sufficient TOA resolution and accuracy even under conditions of considerably lower SNR.

2 System Model

It is assumed that the system is a monostatic radar system and that the temporal distribution of multiple-point scatterers can be expressed as;

$$\theta(t) = \sum_{i=1}^{N_T} a_i \delta(t - \tau_i), \quad (1)$$

where $\delta(*)$ is Dirac's delta function, a_i and τ_i are the i -th scattering coefficient of scatterers and time delay, respectively, and N_T is the number of targets.

The receiving signal $x(t)$ is expressed as;

$$x(t) = \int_{-\infty}^{\infty} \theta(t - \tau)h(\tau)d\tau + n(t), \quad (2)$$

where $h(t)$ is the transmitting signal and $n(t)$ is the thermal noise at the receiver. The discrete form in Eq. (2) is expressed by

$$\mathbf{x} = \mathbf{\Phi}\boldsymbol{\theta} + \mathbf{n}, \quad (3)$$

where $\boldsymbol{\theta} = [\theta(-K\Delta t), \dots, \theta(0), \dots, \theta(N\Delta t)]^T$, $\mathbf{x} = [x(\Delta t), \dots, x(N\Delta t)]^T$, $\mathbf{n} = [n(\Delta t), \dots, n(N\Delta t)]^T$, and

$$\mathbf{\Phi} = \begin{pmatrix} h(K\Delta t) & \dots & h(0) & 0 & \dots & \dots & 0 \\ 0 & h(K\Delta t) & \dots & h(0) & 0 & \dots & 0 \\ & & & \vdots & & & \\ 0 & \dots & \dots & 0 & h(K\Delta t) & \dots & h(0) \end{pmatrix}, \quad (4)$$

where K and N denote the data lengths of the transmitting and receiving signals, respectively. Δt denotes the sampling interval. $\mathbf{\Phi}$ is the observation matrix; and $\boldsymbol{\theta}$ is the actual target distribution. In the case of typical TOA estimation by radar systems, this guarantees that $N_T \ll N$, corresponding to the sparse representation.

3 Conventional Methods

Many studies have been performed aiming to achieve super-resolution TOA estimation. Notably, the MUSIC method [1] exploits the orthogonality between the signal and noise eigenvectors of the correlation matrix to retain super-resolution property beyond the bandwidth. As an alternative approach, CS-based signal decomposition has recently come under the spotlight. The CS-based method achieves accurate signal reconstruction by introducing sparse regularization in the time domain [3].

This is realized by calculating $\boldsymbol{\theta}$ solving the following formula;

$$\hat{\boldsymbol{\theta}} = \arg \min_{\boldsymbol{\theta}} (\|\mathbf{x} - \mathbf{\Phi}\boldsymbol{\theta}\|_2^2 + \lambda\|\boldsymbol{\theta}\|_1), \quad (5)$$

where λ is the regularization coefficient and the $\|\mathbf{v}\|_p$ is l_p norm and denotes $(|v_1|^p + \dots + |v_N|^p)^{\frac{1}{p}}$. While this method achieves super-resolution even for highly correlated signals, and has some advantages relative to the MUSIC based approaches, it still suffers from inaccuracy or degraded resolution in considerably lower SNR situations. This means that the regularization in the time domain is insufficient for suppressing the over-fitting problem in such a case TOA estimation.

4 Proposed Method

To overcome the problem described above, this study introduces frequency domain based regularization into the original CS formulation. It should be noted that is that the target signal should take a sparse distribution not only in the time domain but also in the frequency domain by using much higher

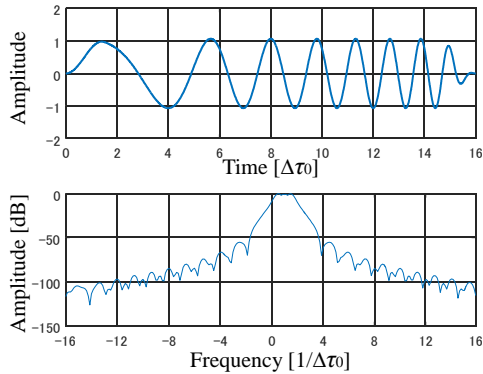


Fig. 1. Waveform of transmitted signal (upper) and Power spectrum of transmitted signal (lower).

A/D conversion than the upper limitation of the Nyquist frequency. In addition, when we have *a priori* knowledge of the maximum frequency of received signals, usually retrieved from the effective bandwidth of the transmitting signal, a sufficient oversampling in the time-domain can be obtained by using zero-padding process in the frequency domain. Then, the dominant ratio of received signal in the frequency domain considerably decreases, namely, a sparsity in the frequency domain is guaranteed. Focusing on this property, the proposed method introduces another regularization term as

$$\hat{\boldsymbol{\theta}} = \arg \min_{\boldsymbol{\theta}} \left(\|\mathbf{x} - \boldsymbol{\Phi}\boldsymbol{\theta}\|_2^2 + \lambda \|\boldsymbol{\theta}\|_1 + \beta \|\mathbf{F}\boldsymbol{\Phi}\boldsymbol{\theta}\|_1 \right), \quad (6)$$

where \mathbf{F} denotes the discrete Fourier transform matrix operator and β is the regularization coefficient. Equation (6) constrains the degree of freedom of reconstructed signals in terms of frequency and time domains. Thus, it prevents the over-fitting to the noise component more strictly than the original CS formulation in Eq. (5).

5 Performance Evaluation Using Numerical Simulation

This section describes the performance evaluation of each method through numerical simulation. Here, the transmitting signal is a chirp-modulated pulse, expressed as

$$h(t) = R(t; T) \exp(j\alpha t^2), \quad (7)$$

$$R(t; T) = \begin{cases} -\cos\left(\frac{\pi}{\tau}t\right) + 1 & (0 \leq t < \tau) \\ 1 & (\tau \leq t < T - \tau) \\ \cos\left(\frac{\pi}{\tau}(t - (T - \tau))\right) + 1 & (T - \tau \leq t \leq T) \\ 0 & (\text{otherwise}) \end{cases}, \quad (8)$$

where α is the chirp rate and T is the pulse length. $\tau = 1.5\Delta\tau_0$, where $\Delta\tau_0$ denotes the time resolution determined by the effective frequency bandwidth of the transmitting signal. Figure 1 shows the assumed transmitting signal in the time and frequency domains. Received signals are generated by Eq. 2

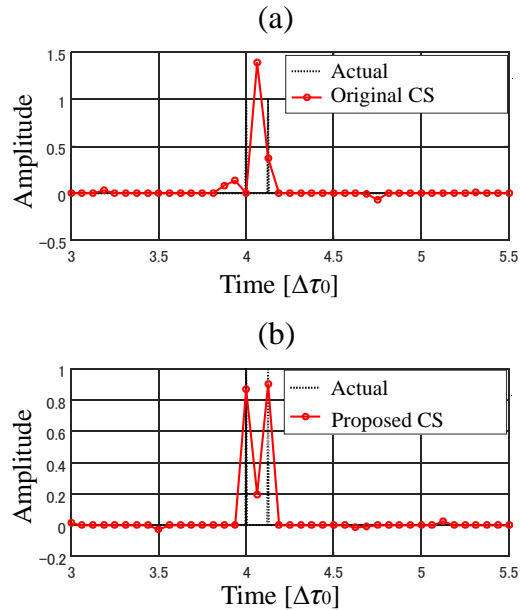


Fig. 2. Reconstruction results at SNR=15dB, in the case of two targets. (a) : The original CS. (b): The proposed methods

with complex white Gaussian noises added as thermal noise $n(t)$. The SNR is defined as the time-averaged power ratio between the signal and noise after applying a bandpass filter determined by the bandwidth of the transmitting signal. The simulation parameters are summarized as follows. The number of targets is 2, the temporal interval of two targets is $\Delta\tau_0/8$, the sampling interval is $\Delta\tau_0/16$, the regularization coefficients are $\lambda = 0.5$, $\beta = 0.01$, and the pulse length is $16\Delta\tau_0$.

Figure 2 shows the reconstruction outputs obtained by the original CS, and the proposed methods, when the mean SNR is around 15 dB. Here, the interior algorithm is used for CS optimization problem, by considering the balance between optimization accuracy and computational cost. The temporal interval is set as $\Delta\tau_0/8$. This figure demonstrates that all methods can decompose highly correlated signals that are adjacent within the temporal resolution. While the original CS methods fail to decompose the two targets, the proposed method maintains the accuracy and resolution, where the two targets are separately decomposed with actual locations.

The RMSE is investigated as reconstruction accuracy evaluation. The RMSE denoted as ϵ and is defined as

$$\epsilon = \sqrt{\frac{\|\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_{\text{true}}\|_2^2}{N_{\text{data}}}}, \quad (9)$$

where $\hat{\boldsymbol{\theta}}$ and $\boldsymbol{\theta}_{\text{true}}$ denote the reconstructed and actual target responses. N_{data} is the data length of the target distribution. Figure 3 plots the median ϵ versus the SNR for each method, with the error bars denoting the interquartile range of ϵ . The number of Monte Carlo trials is 100 in this case. Figure 3

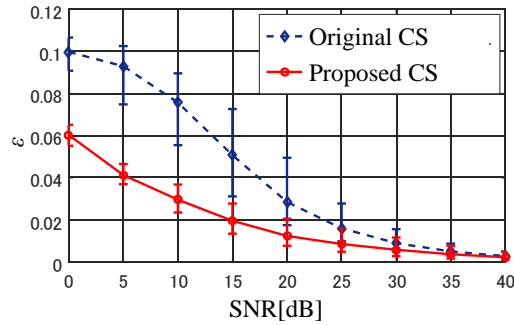


Fig. 3. Median and IQR of ϵ versus SNR for each method in the case of two targets.

demonstrates that the proposed method retains more accurate target reconstruction for not only temporal distribution but also each scattered coefficient. Each calculation time is 17.2 sec for the original CS method and 29.1 sec for the proposed method, respectively, in using an Intel(R) Xeon(R) E5-1620 3.60 GHz processor, where each time value is averaged over 100 trials. The time required for the proposed method is 1.7 times greater than that for the original CS method. The main reason for the higher computational cost is that the proposed method introduces two regularization terms, leading to a sluggish convergence to the optimal solution.

6 Conclusions

Exploiting the sparseness of a received signal spectrum, this study introduced a sparse regularization term in the frequency and time domains to resolve the TOA estimation issue. Numerical simulation results demonstrated that our proposed method maintains the accuracy and super-resolution property even in lower SNR situations, where the completely correlated signals are interfered within an interval that is considerably smaller than the theoretical TOA resolution.