ICA-Based Super Resolution Pulse Compression Algorithm Incorporated by MUSIC Algorithm

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Abstract—**Pulse compression promises higher range resolution in radar systems while avoiding high instantaneous power transmission. For super-high-resolution estimation of the time of arrival, multiple-signal classification (MUSIC) has been already proposed. However, this method suffers severely from accuracy distortion or low resolution when a number of highly correlated interference signals are mixed in the same range gate. As a solution to this problem, we propose a novel pulse compression algorithm by incorporating independent component analysis, which is a powerful tools for blind signal separation, into MUSIC. Numerical simulation shows that the proposed method achieves higher range resolution and accuracy than conventional MUSIC, particularly for lower signal-to-noise ratios.**

Key words—**Independent component analysis (ICA), multiple-signal classification (MUSIC), radar pulse compression, time of arrival (TOA) estimation.**

I. INTRODUCTION

Pulse compression is commonly used in radar systems to obtain a sufficient range resolution while avoiding high peak transmission power. As the classical pulse compression scheme, cross-correlation algorithms [1], [2] have been intensively employed. However, such algorithms hardly separate the target signal within the same range gate strictly determined by the transmission bandwidth. As a super-high-resolution technique to resolve the above difficulty, multiple-signal classification (MUSIC) [3], which is based on the resolution exploiting the eigenvectors of the noise subspace, has been proposed. However, its accuracy in estimating time of arrival (TOA) seriously degrades for lower signal-to-noise ratios (SNRs) or in the case that there are a number of highly correlated interference signals in the same range gate.

To overcome the above problem, this paper proposes a novel pulse compression algorithm by incorporating independent component analysis (ICA) into MUSIC. ICA is one of the most useful tools for blind source separation, and requires only the statistical independence of the source signals for signal reconstruction [4]. In recent years, several ICA algorithms suitable for complex signals have been developed [5], [6], and successfully decomposed multiple deterministic signals as complex sinusoidal signals with different frequencies have been reported [7]. Such algorithms are also useful for pulse compression, because TOA estimation corresponds to the separation of the multiple complex sinusoidal signals in the frequency domain. As preprocessing for the proposed method, ICA is applied to observation signals to avoid resolution degradation due to coherent interference effects.

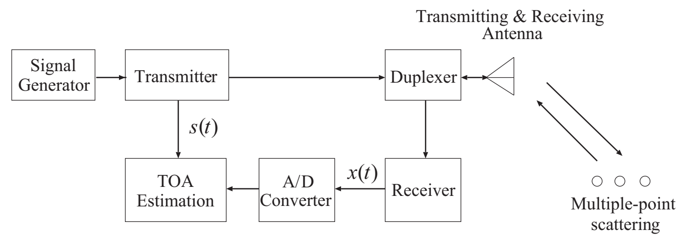

Fig. 1. System model.

In addition, to enhance the separation performance of complex sinusoidal signals at the frequency resolution, we introduce a novel ICA algorithm based on maximum likelihood criteria specified by *a priori* information of the probability density function (PDF) of complex sinusoidal signals. Numerical simulations verify that the range resolution and accuracy of the proposed method are superior to those of the conventional MUSIC method.

II. SYSTEM MODEL

Figure 1 shows the system model. This paper assumes monostatic radar and multiple-point scattering. We consider a chirp-modulated transmitted signal, and the received signal is simply expressed as

$$x(t) = \sum_{i=1}^{L} a_i s(t - \tau_i) + n(t), \tag{1}$$

where a_i is amplitude, τ_i denotes each time of arrival, L is the number of signals, $s(t)$ denotes the transmitted signal with chirp modulation, $n(t)$ expresses a Gaussian white noise. We define the frequency transfer function as

$$Z(n\Delta\omega) \equiv \frac{X(n\Delta\omega)}{S(n\Delta\omega)} = \sum_{i=1}^{L} a_i \exp(-\mathrm{j}n\Delta\omega\tau_i) + \frac{N(n\Delta\omega)}{S(n\Delta\omega)}, \quad (n = 1, 2, ..., N), \tag{2}$$

where $X(\omega)$, $S(\omega)$ and $N(\omega)$ are the Fourier transform of $x(t)$, $s(t)$ and $n(t)$, N is the total number of frequency points, and $\Delta\omega$ denotes the frequency sampling rate.

III. CONVENTIONAL METHOD

MUSIC has been proposed as a super-high-resolution method for TOA estimation. MUSIC positively employs eigenvectors determined by the noise subspace decomposed from

$Z(n\Delta\omega)$. First, the correlation matrix is calculated as

$$\boldsymbol{R} = \sum_{n=1}^{N-M+1} \boldsymbol{Z}_n \boldsymbol{Z}_n^H, \tag{3}$$

where ^{H} denotes the Hermitian transpose, $\boldsymbol{Z}_n = [Z(n\Delta\omega), \cdots, Z((n+M-1)\Delta\omega)]^T$, and $M(<N)$ denotes the dimension of the subspace in the frequency domain. The output of conventional MUSIC is

$$y_{\mathrm{music}}(t) = \frac{\boldsymbol{a}^H(t)\boldsymbol{a}(t)}{\boldsymbol{a}^H(t)\boldsymbol{E}_n{\boldsymbol{E}_n}^H\boldsymbol{a}(t)}, \tag{4}$$

where $\boldsymbol{a}(t) = [\exp(-\mathrm{j}\Delta\omega t), \cdots, \exp(-\mathrm{j}M\Delta\omega t)]^T$ is called steering vector, $\boldsymbol{E}_n = [\boldsymbol{e}_{\hat{L}+1}, \cdots, \boldsymbol{e}_{N-M-\hat{L}+1}]$ is the eigenvalue expansion of $\boldsymbol{R}$, and $\hat{L}$ is the estimated number of signals. The TOA is obtained from the local maximum of $y_{\mathrm{music}}(t)$. Although frequency averaging is applied to suppress resolution degradation due to coherent interference signals, there is still accuracy distortion as the number of such signals and noise components increase.

IV. Proposed Method

As a solution to the aforementioned problem, this paper proposes a novel pulse compression algorithm by incorporating ICA into MUSIC. First, we briefly explain the original ICA algorithm based on maximum likelihood criteria (MLICA) [6]. ICA requires only the statistical independence of source signals for separation and no prior information of desired signals, where uncertainty in the scale and permutation shifts are allowed. While a typical ICA model requires multiple channels for the separation of multiple sources, our system model assumes that the observed signal is received in only a single channel. As a solution, single-channel ICA (SCICA) [8] is adopted in this study. SCICA creates quasi multiple channels employing a frequency shift of the received signal. The SCICA model creates L' the quasi multiple channel as

$$\boldsymbol{Z} = [\boldsymbol{Z}_1, \boldsymbol{Z}_2, \cdots, \boldsymbol{Z}_{L'}]^T$$

$$\boldsymbol{Z}_n = [Z(n\Delta\omega), Z((n+1)\Delta\omega), \cdots, Z((n+N-L')\Delta\omega)] \quad (n = 1, 2, \cdots, L') \tag{5}$$

The uncorrelated signals after PCA (Principal Component Analysis) is formulated as

$$\boldsymbol{Z}^P = \boldsymbol{M}\boldsymbol{Z}, \tag{6}$$

where $\boldsymbol{M}$ is $\hat{L}' \times L'$ matrix, which is computed the singular value decomposition of $\boldsymbol{Z}$, and $\hat{L}'$ is estimated number of signals. The reconstruction through ICA is then formulated as

$$\hat{\boldsymbol{Z}} = \boldsymbol{W}\boldsymbol{Z}^P, \tag{7}$$

where $\boldsymbol{W}$ is a reconstruction matrix. According to the definition given as Eq. (2), TOA estimation is equivalent to the separation of multiple complex sinusoidal signals. Several ICA algorithms suitable for this issue have been proposed on the basis of maximizing the non-Gaussianity [5] or the likelihood criteria [6]. Since these ICA algorithms require *a priori* knowledge of the PDF of source signals, the separation performance strongly depends on the selected PDF. Fortunately, in this case, the PDF of the source signal, which is a sinusoidal wave, is known, and it can be used to enhance separation performance. In the proposed method, MLICA is extended to employ the PDF of the sinusoidal signal, and it updates the reconstruction matrix $\boldsymbol{W}$ by maximizing the likelihood function [6]:

$$\boldsymbol{W} \leftarrow \boldsymbol{W} + \mu\left(\boldsymbol{I} - \langle\psi\left(\hat{\boldsymbol{Z}}\right)\hat{\boldsymbol{Z}}^H\rangle\right)\boldsymbol{W}, \tag{8}$$

where $\boldsymbol{I}$ is an unit matrix, μ is a learning coefficient, $<*>$ denotes an ensemble averaging. The score function defined as $\boldsymbol{\psi}(\hat{\boldsymbol{Z}}) \equiv [\psi(\hat{\boldsymbol{Z}}_1), ..., \psi(\hat{\boldsymbol{Z}}_{\hat{L}'})]^T$ is calculated as

$$\psi(\hat{\boldsymbol{Z}}_l) = \frac{1}{2}\left(\frac{\partial \log p_s(\hat{\boldsymbol{Z}}_l^r, \hat{\boldsymbol{Z}}_l^i)}{\partial \hat{\boldsymbol{Z}}_l^r} + \mathrm{j}\frac{\partial \log p_s(\hat{\boldsymbol{Z}}_l^r, \hat{\boldsymbol{Z}}_l^i)}{\partial \hat{\boldsymbol{Z}}_l^i}\right), \tag{9}$$

where $\hat{\boldsymbol{Z}}_l^r$, $\hat{\boldsymbol{Z}}_l^i$ is real and imaginary part of $\hat{\boldsymbol{Z}}_l$ and p_s is PDF of source signals. Here, we assume that the real and imaginary parts of complex sinusoidal signals are statistically independent. The PDF of a complex sinusoidal signal is given by

$$p_s\left(\hat{Z}_l\right) = \begin{cases} \frac{1}{\pi^2\sqrt{\left(1-\hat{Z}_l^{r\,2}\right)\left(1-\hat{Z}_l^{i\,2}\right)}} & \left(|\hat{Z}_l^r|, |\hat{Z}_l^r| < 1\right), \\ 0 & (\text{otherwise}), \end{cases} \tag{10}$$

TABLE I
Actual Procedure of the Proposed ICA Algorithm.

1) Set an initial reconstruction matrix $\boldsymbol{W}$ and learning rate μ.
2) Calculate $\hat{\boldsymbol{Z}} = \boldsymbol{W}\boldsymbol{Z}^P$.
3) Update the reconstruction matrix by
$$\boldsymbol{W} \leftarrow \boldsymbol{W} + \mu\left(\boldsymbol{I} - E\left[\hat{\boldsymbol{\psi}}\left(\hat{\boldsymbol{Z}}\right)\hat{\boldsymbol{Z}}^H\right]\right)\boldsymbol{W}.$$
4) Go back to step 3 until $\| \boldsymbol{W}(t) - \boldsymbol{W}(t-1) \| < J$ is satisfied.

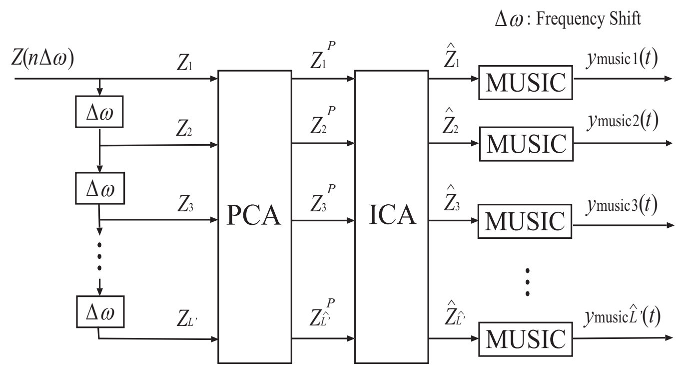

Fig. 2. Block diagram of the proposed method.

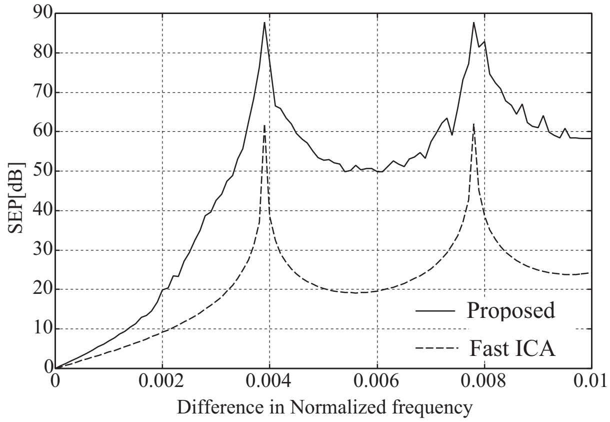

Fig. 3. Separation performance with each algorithm versus normalized frequency difference.

From Eqs. (9) and (10), when $|\hat{Z}_l^r|, |\hat{Z}_l^i| < 1$ holds, the score function ψ defined as

$$\psi\left(\hat{Z}_l\right) = \frac{1}{2}\left(\frac{\hat{Z}_l^r}{1-{\hat{Z}_l^r}^2} + \mathrm{j}\frac{\hat{Z}_l^i}{1-{\hat{Z}_l^i}^2}\right). \tag{11}$$

It should be noted that ψ diverges to infinity when $|\hat{Z}_l^r|, |\hat{Z}_l^i| \rightarrow 1$, and $\boldsymbol{W}$ in Eq. (8) tends to deviate from the optimum solution. To mitigate this divergence, a cushion factor is introduced:

$$\hat{\boldsymbol{\psi}}\left(\hat{\boldsymbol{Z}}_l\right) = r\boldsymbol{\psi}\left(\hat{\boldsymbol{Z}}_l\right). \tag{12}$$

Table I presents the actual procedure of the proposed ICA algorithm. The learning process continues until $\| \boldsymbol{W}(t) - \boldsymbol{W}(t-1) \| < J$ is satisfied, where J is empirically determined. However, the separation performance of this ICA algorithm strongly depends on the cushion factor. Here, we focus on the fact that the maximum value of the likelihood function is strongly correlated to the separation performance. The optimal cushion factor $r_{\rm opt}$ is then determined as

$$r_{\rm opt} = \mathop{\arg\max}_{0<r<1} \sum_{k=1}^{T}\sum_{l=1}^{\hat{L}'} \log \frac{1}{\pi^2\sqrt{(1-\hat{Z}_l^r(k)^2)(1-\hat{Z}_l^i(k)^2)}} + T\log|\det \boldsymbol{W}|, \tag{13}$$

where T is the total data length. The trust-region algorithm [9] is used to solve the optimization problem.

Figure 2 is a block diagram of the proposed method. The quasi multiple channels are created with a frequency shift. Next, PCA is applied to $\boldsymbol{Z}$ to obtain uncorrelated signals as $\boldsymbol{Z}^P$, where the observation noise is considerably suppressed. Applying the proposed MLICA algorithm to $\boldsymbol{Z}^P$, the reconstruction signal $\hat{\boldsymbol{Z}}$ is obtained. Finally, MUSIC is applied to each channel of the reconstruction signal $\hat{\boldsymbol{Z}}$. Each TOA is estimated from the maximum time of the MUSIC output. The main advantage of this algorithm is that highly correlated signals are decomposed by ICA in each channel of $\hat{\boldsymbol{Z}}$ and the range resolution of the conventional MUSIC is thus enhanced.

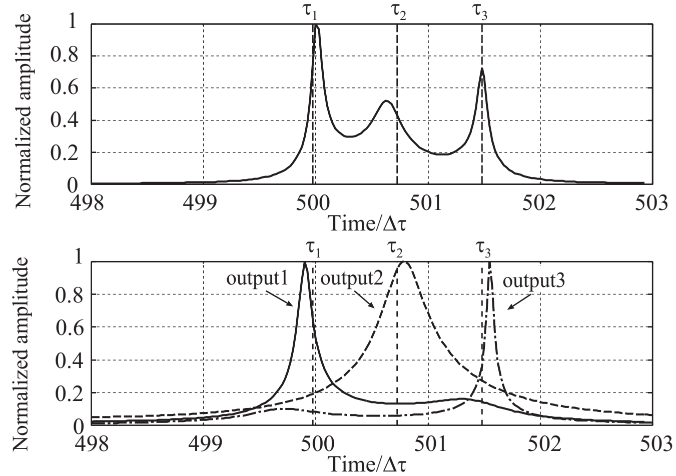

Fig. 4. Outputs of MUSIC (upper) and proposed method (lower) at SNR=30dB.

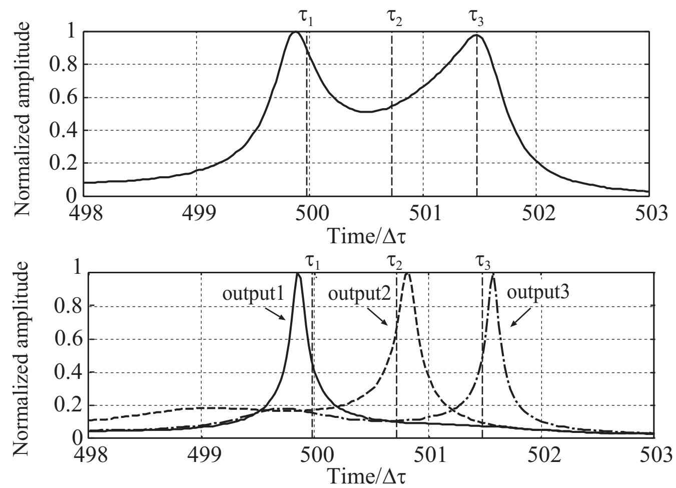

Fig. 5. Outputs of MUSIC (upper) and proposed method (lower) at SNR=20dB.

V. Performance Evaluation in Numerical Simulation

This section evaluates the accuracies of the conventional and proposed methods in numerical simulations. First, the separation performance of the proposed MLICA algorithm is assessed to clarify its superiority over the conventional MLICA algorithm. Two complex sinusoidal signals with different frequencies are created, and the observed signal is generated using a proper mixing matrix. The separation performance SEP is defined as the ratio of the desired-signal power to the unwanted-signal power for each separated signal [7]. Figure 3 shows SEP for the frequency difference in a noiseless environment where the frequency is normalized by the maximum frequency determined by the data length $N = 256$. The broken line shows SEP obtained by FastICA on the basis of maximizing non-Gaussianity [5] and the solid line presents

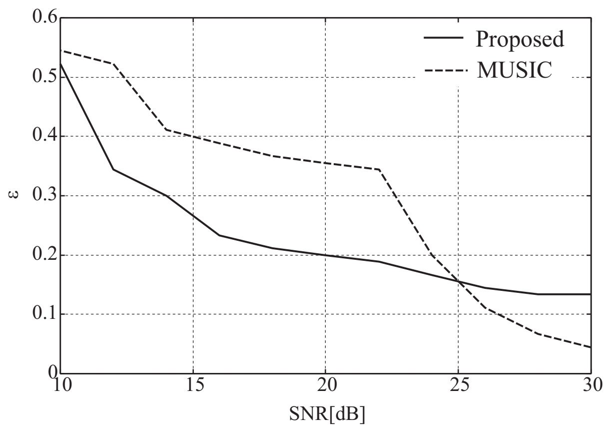

Fig. 6. TOA errors against the SNR for each method.

that of the proposed ICA algorithm. The figure shows that the proposed method has much higher separation performance than the conventional ICA algorithm at all frequency differences because *a priori* knowledge of the PDF offers a more accurate solution for ICA optimization.

Next, the performance of TOA estimation in pulse compression is presented. The upper and lower graphs in Figs. 4 and 5 respectively show the outputs of conventional MUSIC and the proposed method for SNRs of 30 and 20 dB. Here, the number of signals is $L = 3$, and is the TOAs are set as $\tau_1 = 499.975\Delta\tau$, $\tau_2 = \tau_1 + 0.75\Delta\tau$, $\tau_3 = \tau_2 + 0.75\Delta\tau$, where $\Delta\tau$ denotes the time resolution determined by the frequency band width of the transmitted signal, and amplitude of signals are the same as $a_i = 1$. White Gaussian noise is added to the observed signals. Here, the SNR is defined as the ratio of the peak signal power to the average noise power in the time domain. The figures show that conventional MUSIC and the proposed method both produce significant peaks around all actual TOAs at an SNR of 30 dB. By contrast, at an SNR of 20 dB, conventional MUSIC suffers from insufficient range resolution and cannot obtain the TOA around $t = \tau_2$. This is because uniform frequency averaging is not enough to suppress coherent interference signals and noisy components. On the contrary, the proposed method produces three local peaks around the actual TOA; that is, higher range resolution is achieved. This is because the proposed method considerably suppresses the coherent signal in MLICA processing, specifying the sinusoidal wave separation.

Finally, the quantitative analysis for each method is investigated. Here the normalized accuracy ϵ is defined as

$$\epsilon = \frac{1}{L}\sum_{i=1}^{L} \min_j|\tau_i - \hat{\tau}_j|, \tag{14}$$

where $\hat{\tau}_j$ is estimated TOA. Figure 6 shows the normalized accuracy of TOA estimation against the SNR for each method. The number of trials is 100. The figure shows that the proposed method accomplishes more accurate TOA estimation for a lower SNR. In particular at an SNR of 20 dB, the normalized accuracy for our method is 1.7 times that for MUSIC. This can be reasoned by the fact that the proposed method suppresses Gaussian white noise through decorrelation processing employing PCA, which is not used in the conventional MUSIC algorithm. However, TOA estimation by conventional MUSIC is better than that for the proposed method at a higher SNR. This is because the effective frequency bandwidth of the transfer function is narrower when employing the proposed method owing to the frequency shift in SCICA processing.

VI. Conclusion

This paper proposed a novel approach for super-high-resolution TOA estimation employing ICA as preprocessing for MUSIC. To enhance the range resolution and accuracy of conventional MUSIC in situations of severe interference or noise, the extended MLICA algorithm for specifying the separation of complex sinusoidal signals was established, and incorporated into MUSIC. Numerical simulations verified that the range resolution and accuracy of the proposed method, especially those at lower SNRs, were superior to those of conventional MUSIC. It is our future work to validate the proposed algorithm using experimental data.

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