ICA Algoritm with Likelihood Criterion to Separate Mixtures of Complex Sinusoidal Signals

Tetsuhiro Okano Shouhei Kidera Tetsuo Kirimoto

Graduate School of Informatics and Engineering, University of Electro-Communications, Tokyo, Japan, okano@secure.ee.uec.ac.jp

Abstract: In this paper we consider the blind source separation (BSS) of complex sinusoidal signals with different frequencies. We introduce a novel ICA (Independent Component Analysis) algorithm for the BSS. ICA requires no prior information of the source signals because it employs only the statistical independence of them. We have already confirmed that ICA was successfully applied to a deterministic signal like a complex sinusoidal signal. However, the frequency resolution of the former algorithm is not sufficient for the actual radar applications, such as pulse compression or clutter refection. To enhance frequency resolution, we propose a novel ICA algorithm in specifying the complex sinusoidal signal separation using the likelihood criterion. The conventional maximum likelihood based ICA algorithm typically selects the PDF (probability density function) from some promising candidates. Thus, we introduce the PDF of a complex sinusoidal signal. The results in numerical simulations verify that the proposed method successfully separates the multiple sinusoidal signals with frequencies close to less than DFT (Discrete Fourier Transform) resolution.

Key words: Independent component analysis (ICA), Blind source separation (BSS), Complex sinusoidal signals, Separation performance, Radar signal processing.

1 Introduction

BSS is the approach to estimate source signals using only the information of the mixed signals observed in multiple channel input. As one of the promising BSS solution, many kinds of ICA algorithms [1][2] have been employed in various fields of signal processing as an acoustic or an electromagnetic signal. ICA requires only a statistical independence of source signals for separation and no prior information of desired signals, where the uncertainty for the scale and permutation shifts are allowed. However, ICA assumes that source signals are statistical signals, not deterministic signals such as complex sinusoidal signals. It is absolutely essential that ICA can separate mixture of deterministic signals to apply some applications such as a pulse compression in radar or communication systems. We have already verified that ICA could be successfully applied to deterministic singles such as complex sinusoidal signals with different frequencies [3]. However, in order to apply ICA in radar signal processing such as pulse compression and clutter suppression, it is desired to improve the separation performance of complex sinusoidal signals in ICA. This is because the frequency resolution of the former algorithm is not sufficient for actual radar applications.

In order to solve above problems, we propose a novel ICA algorithm using maximum likelihood scheme, which utilizes the PDF of each source signals. Typically in maximum likelihood scheme in ICA, the form of the PDF is chosen from some promising candidates [4]. Then, the frequency resolution seriously depends on the selected PDF. In this paper, we employ the PDF of complex sinusoidal signals to enhance the frequency resolution. The results in numerical simulations verify that separation performance of the proposed algorithm is superior to that of the conventional ICA algorithm. The proposed method shows that it can considerably reduce the lower limit of frequency resolution.

2 Observation Model

This paper deals with complex sinusoidal signals with different frequencies. The source signals are $s_1(t) = \exp(\mathrm{j}2\pi f_1 t)$, $s_2(t) = \exp(\mathrm{j}2\pi f_2 t)$, where t is time, f is frequency and denote the $f_1 \neq f_2$. ICA model assumes that the observed signals are expressed as a linear mixture of the source signals. Then, observed signals are given by

$$\boldsymbol{x} = \boldsymbol{B}\boldsymbol{s} + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{x} = \left[x_1(t), x_2(t)\right]^T$, $\boldsymbol{s} = \left[s_1(t), s_2(t)\right]^T$, $\boldsymbol{B}$ is an unknown 2×2 matrix called mixing matrix, and $\boldsymbol{n}$ is a observation noise. Here, PCA (Principal Component Analysis) is applied to $\boldsymbol{x}$ in order to obtain uncorrelated signals as $\hat{\boldsymbol{x}}$. Using whitening matrix $\boldsymbol{M}$, $\hat{\boldsymbol{x}}$ is expressed as $\hat{\boldsymbol{x}} = \boldsymbol{M}\boldsymbol{x}$. ICA estimates source signals as $\boldsymbol{u}$ using a reconstruction matrix $\boldsymbol{W}$ as

$$\boldsymbol{u} = \boldsymbol{W}\boldsymbol{x}. \tag{2}$$

3 Conventional Method

In this section, we explain the conventional ICA algorithms as FastICA and Maximum likelihood ICA. First, the Fast ICA algorithm calculates kurtosis of the separated signals and find out reconstruction matrix $\boldsymbol{W}$ according to maximizing the non-Gaussianity. The update rule for the algorithm is given by [5]

$$\boldsymbol{W} \leftarrow \langle \boldsymbol{x} \left(\boldsymbol{W}^H \boldsymbol{x}\right)^* g \left(\mid \boldsymbol{W}^H \boldsymbol{x} \mid^2\right)\rangle$$
$$- \langle g \left(\mid \boldsymbol{W}^H \boldsymbol{x} \mid^2\right) + \mid \boldsymbol{W}^H \boldsymbol{x} \mid^2 g' \left(\mid \boldsymbol{W}^H \boldsymbol{x} \mid^2\right)\rangle \boldsymbol{W}, \tag{3}$$

$$\boldsymbol{W} \leftarrow \boldsymbol{W} / \parallel \boldsymbol{W} \parallel, \tag{4}$$

where $\langle\cdot\rangle$ denotes time average, g can be any suitable non-quadratic contrast function, g' denotes a derivative of g, and * is complex conjugate. Because the Fast ICA is based on maximization of non-Gaussianity for measuring the statistical independence, it is difficult to maintain frequency resolution.

On the contrary, the maximum likelihood ICA utilizes PDF of source signals, and update reconstruction matrix W by maximizing the following likelihood function as

$$L(\boldsymbol{W}) = \log p_s(\boldsymbol{W}\boldsymbol{x}) + \log \mid \det \boldsymbol{W} \mid, \tag{5}$$

where p_s is PDF of source signals. This method yields the relative natural gradient updates in optmizing $L(\boldsymbol{W})$, then update rule is expressed as [6]

$$\boldsymbol{W} \leftarrow \boldsymbol{W} + \mu \left(\mathbf{I} - \langle \psi(\boldsymbol{u}) \boldsymbol{u}^H \rangle\right) \boldsymbol{W}, \tag{6}$$

where ^{H} denotes Hermitian conjugate, $\mathbf{I}$ is unit matrix and μ is learning coefficient. Score function ψ is calculated as

$$\psi(\boldsymbol{u}) = \frac{1}{2}\left(\frac{\partial \log p_s(\boldsymbol{u}_r, \boldsymbol{u}_i)}{\partial \boldsymbol{u}_r} + \mathrm{j}\frac{\partial \log p_s(\boldsymbol{u}_r, \boldsymbol{u}_i)}{\partial \boldsymbol{u}_i}\right), \tag{7}$$

where $\boldsymbol{u}_r$, $\boldsymbol{u}_i$ are the real and complex part of $\boldsymbol{u}$. A maximum likelihood scheme of ICA estimates both reconstruction matrix $\boldsymbol{W}$ and the nonlinearity of PDF of each source signals. Typically in maximum likelihood ICA, the form of the PDF is chosen from the sub-gaussian or super-gaussian nature of the estimated source signals [4]. Therefore, we need to select the PDF from some promising candidates.

4 Proposed Method

To solve the problem described in the avobe, we incorporate the PDF of the complex sinusoidal signal in the maximum likelihood ICA as prior information. This is because general radar application is required separation mixtures of complex sinusoidal signals with different frequencies and the PDF of them should be known. Here, we assume that real and imaginary part of complex sinusoidal signals is statistical independence. The PDF of complex sinusoidal signal is given by the following equations

$$p_s(u) = \begin{cases} \frac{1}{\pi^2\sqrt{\left(A^2-u_r^2\right)\left(A^2-u_i^2\right)}} & (|u_r|, |u_i| < A), \\ 0 & (\text{otherwise}), \end{cases} \tag{8}$$

where A is constant. Using Eq. (7), the score function $\psi(\boldsymbol{u})$ is derived as

$$\psi(u) = \frac{1}{2}\left(\frac{u_r}{A^2 - {u_r}^2} + \mathrm{j}\frac{u_i}{A^2 - {u_i}^2}\right). \tag{9}$$

Table 1: Actual procedure of the proposed method.

1. Apply PCA to the observed signals as $\hat{\boldsymbol{x}} = \boldsymbol{M}\boldsymbol{x}$
2. Set an initial reconstruction matrix $\boldsymbol{W}$ and learning rate μ.
3. Calculate $\boldsymbol{u} = \boldsymbol{W}\hat{\boldsymbol{x}}$
4. Update the reconstruction matrix by
$$\boldsymbol{W} \leftarrow \boldsymbol{W} + \mu\left(\mathbf{I} - E\left[\hat{\psi}(\boldsymbol{u})\boldsymbol{u}^H\right]\right)\boldsymbol{W}$$
5. Go back to step 3 until $\parallel \boldsymbol{W}(t) - \boldsymbol{W}(t-1) \parallel < J$ is satisfied.

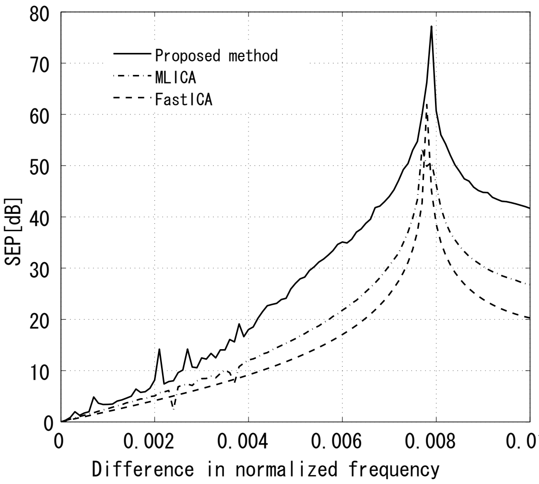

Fig. 1: Separation performance with each algorithm for different frequencies (N=128).

It should be noted that $\psi(\boldsymbol{u})$ diverge to infinity when $|u_r|, |u_i| \rightarrow A$, and Eq. (6) does not converge to an appropriate solution. To control this acute change, we introduce

$$\hat{\psi}(\boldsymbol{u}) = r\psi(\boldsymbol{u}), \tag{10}$$

where r is empirically determined called cushion factor. Table. 1 the actual procedure of the proposed method. The learning process is continued until $\parallel \boldsymbol{W}(t) - \boldsymbol{W}(t-1) \parallel < J$ is satisfied, where J is empirically determined.

5 Performance Evaluation in Numerical Simulation

In this section we evaluate the separation performance of proposed method using numerical simulation. We create the two complex sinusoidal signals with different frequencies using proper mixing matrix, and generate observed signals. Separation performance is evaluated by

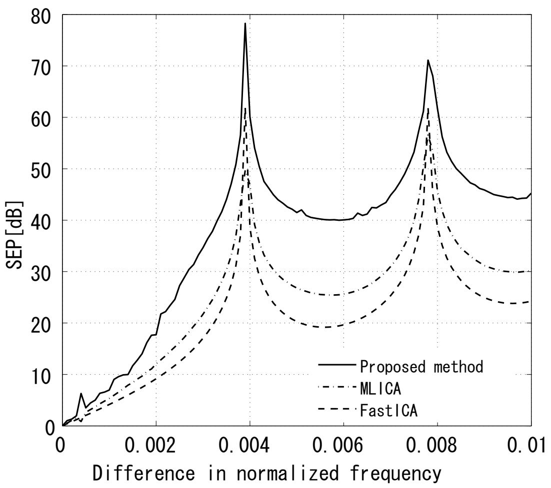

Fig. 2: Separation performance with each algorithm for different frequencies (N=256).

the following equation [7]

$$\mathrm{SEP}\,(i) = \frac{1}{N}\sum_{i=1}^{\mathrm{N}} \frac{\max_j\left(|P_{ij}|^2\right)}{\sum_{j=1}^{N}|P_{ij}|^2 - \max_j\left(|P_{ij}|^2|\right)}, \quad (11)$$

where $\boldsymbol{P} = \boldsymbol{WMB}$ is a 2×2 complex matrix, and P_{ij} denotes the element of $\boldsymbol{P}$ at the i the row and the j th column. SEP becomes infinity when the ICA completely separates the observed signals. SEP also qualitatively means the ratio of the desired and unnecessary signal powers contained in each separated signals.

Figs. 1 and 2 show SEP for the frequency difference, which is normalized by the inverse of data length N as 128 and 256 respectively, where broken line is FastICA based on maximizing non-Gaussianity, chain line is maximum likelihood ICA (MLICA) using PDF of complex sinusoidal signal and solid line is proposed method at cushion factor $r = 0.2$. As shown in this figure, the proposed method obtains much higher separation performance than conventional method in all frequency differences. Fourier analysis can separate observed signals when difference in normalized frequency is more than 1/N. Fig. 3 show real part of observed signal waveforms and separated signal waveforms with the conventional and the proposed methods, respectively when difference in the frequency difference is 0.0045 and data length is 128. Each of SEP with FastICA, MLICA and proposed method is 10.7 dB, 13.9 dB and 22.2 dB, respectively. As shown in this figure, separate signals waveform with FastICA and MLICA are imperfectly separated. On the contrary, the proposed method successfully separates the two original signals, and it verifies that this method improves the frequency resolution in this case. Here, we define frequency resolution for each method, where its SEP is greater than 20 dB. Table. 2 shows comparison of frequency resolution for each algorithm when data length N are 128 and 256. This table shows that the proposed method offers much higher frequency resolution than that of the other conventional methods. We evaluate the performance of the proposed method in noisy environment. We add white gaussian noise to observed signals. Fig. 4 shows separation performance with each algorithm for different frequencies when SNR (Signal-to-Noise Ratio) is 30dB. As shown in this figure, proposed method provides better separation performance than conventional ICA algorithms.

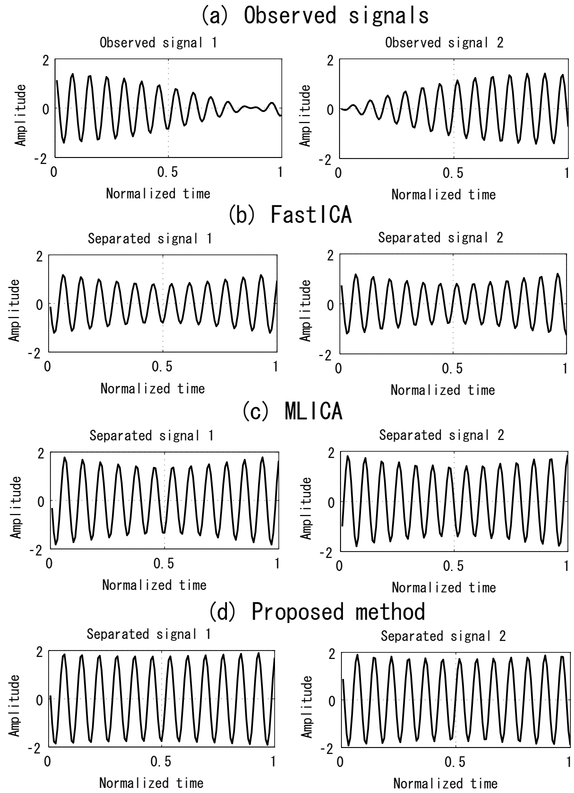

Fig. 3: Real part of waveform. (a) observed signal waveforms and separated signals with (b) FastICA (c) MLICA (d) the proposed method.

Table 2: Comparison of frequency resolution.

frequency resolution	Fourier analysis	Fast ICA	ML ICA	Proposed method
f_{128}	0.0078	0.0065	0.0058	0.0042
f_{256}	0.0039	0.0032	0.0029	0.0021

6 Conclusion

In this paper, we proposed a novel ICA algorithm in specifying the separation of complex sinusoidal signals with different frequencies. The conventional ICA algorithm using only information of observed signals, result in frequency resolution degradation. To solve this problem, we incorporated PDF of complex sinusoidal signal in ICA algorithm based on likelihood criterion. We then show that the algorithm modifies the score function to be suitable for the complex sinusoidal signal. The results

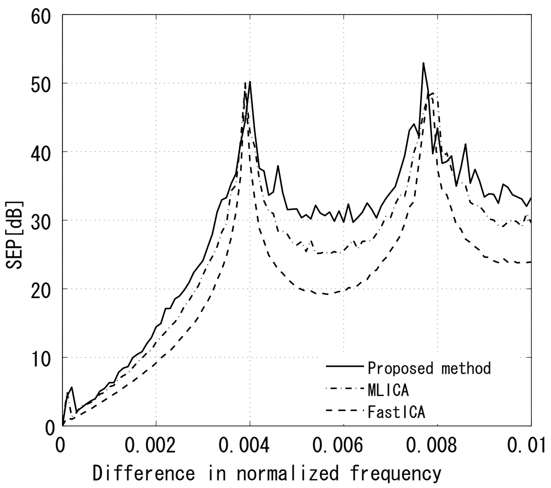

Fig. 4: Separation performance with each algorithm for different frequencies (N=256) for SNR=30dB.

in numerical simulations verified that separation performance of proposed algorithm, especially for the multiple complex sinusoidal signals with frequencies close to less than DFT resolution, is superior to that of the conventional ICA algorithm.

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